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13. ABSTRACT *(Maximum 200 words)*

Thin-walled structures of interest to the U. S. Air Force, such as aircraft fuselages, rocket casings, helicopter blades, and containment vessels, are often constructed of layers of anisotropic, filament or fiber-reinforced materials which must be designed to remain elastic. Our research has been concerned with load diffusion in such structures. An understanding of the fundamental mechanisms of load diffusion in composite subcomponents is essential in developing primary composite structures. Analytical models of load diffusion behavior are extremely valuable in building an intuitive base for developing refined modeling strategies and assessing results from finite element analyses. The decay behavior of stresses and other field quantities provides a significant aid towards this process. Our results are also useful for structural tailoring.

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FINAL TECHNICAL REPORT

1. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER AFOSR (AASERT) SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL REFERENCES:

   (i) Anti-plane shear deformations of anisotropic sandwich structures: end effects (Sarah C. Baxter and C. O. Horgan), *Int. J. of Solids and Structures,* 34, 1997, 79-98.

   (ii) End effects for anti-plane shear deformations of sandwich structures (with Sarah C. Baxter), *J. of Elasticity,* 40, 1995, 123-164.

   (iii) Effects of curvilinear anisotropy on radially symmetric stresses in anisotropic linearly elastic solids (C. O. Horgan and S. C. Baxter), *J. of Elasticity,* 42, 1996, 31-48.

   (iv) Saint-Venant edge effects in sandwich structures (C. O. Horgan and Sarah C. Baxter), Proceedings of the 14th U.S. Army Symposium on Solid Mechanics, Myrtle Beach, S.C., October 1996 (ed. by K. R. Iyer and S. C. Chou), Battelle Press, Columbus, 1997, pp. 652-661.

   (v) Saint-Venant's principle for sandwich structures (C. O. Horgan and S. C. Baxter), MECHANICS OF SANDWICH STRUCTURES (Ed. by A. Vautrin), Proceedings of the EUROMECH 360 Symposium on "Mechanics of Sandwich Structures," St. Etienne, France, May 1997 , Kluwer, Dordrecht, 1998, pp.113-122.

   (vi) Saint-Venant decay rates for an isotropic inhomogeneous linearly elastic solid in anti-plane shear (M. R. Scalpato and C. O. Horgan) *J. of Elasticity* 48, 1997, 145-166.

   (vii) End effects in anti-plane shear of an inhomogeneous isotropic linearly elastic semi-infinite strip (Alice M. Chan and C. O. Horgan), submitted

2. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:
   Sarah C. Baxter (Ph.D. Degree, 1995); Monica R. Scalpato (M.S. Degree, 1997); Alice M. Chan (M.S. expected May 1999): Adam J. Lewandowski (B.S Degree, 1998).

3. REPORT OF INVENTIONS (BY TITLE ONLY): None

4. *SCIENTIFIC PROGRESS AND ACCOMPLISHMENTS*

Preliminary remarks:

Thin-walled structures of interest to the U. S. Air Force, such as aircraft fuselages, rocket casings, helicopter blades, and containment vessels, are often constructed of layers of anisotropic, filament or fiber-reinforced materials which must be designed to remain elastic. Our research has been concerned with load diffusion in such structures. An understanding of the fundamental mechanisms of load diffusion in composite subcomponents is essential in developing primary composite structures. Analytical models of load diffusion behavior are extremely valuable in building an intuitive base for developing refined modeling strategies and assessing results from finite element analyses. The decay behavior of stresses and other field quantities provides a significant aid towards this process. Our results are also useful for structural tailoring.

Summary of findings:

In (i), (ii), (iv), (v) the purpose is to further investigate the effects of material inhomogeneity and the combined effects of material inhomogeneity and anisotropy on the decay of Saint-Venant end effects. Saint-Venant decay rates for self-equilibrated edge loads in symmetric sandwich structures are examined in the context of anti-plane shear for linear anisotropic elasticity. The problem is governed by a second-order, linear, elliptic partial differential equation with discontinuous coefficients. The most general anisotropy consistent with a state of anti-plane shear is considered, as well as a variety of boundary conditions. Anti-plane or longitudinal shear deformations are one of the simplest classes of deformations in solid mechanics. The resulting deformations are completely characterized by a single out-of-plane displacement which depends only on the in-plane coordinates. They can be thought of as complementary deformations to those of plane elasticity. While these deformations have received little attention compared with the plane problems of linear elasticity, they have recently been investigated for *anisotropic* and *inhomogeneous* linear elasticity. In the context of linear elasticity, Saint-Venant's principle is used to show that self-equilibrated loads generate local stress effects which quickly decay away from the loaded end of a structure. For homogeneous *isotropic* linear elastic materials this is well-documented. Self-equilibrated loads are a class of load distributions that are statically equivalent to zero, i.e., have zero resultant force and moment. When Saint-Venant's principle is valid, pointwise boundary conditions can be replaced by more tractable resultant conditions. It is shown in the above references that material inhomogeneity significantly affects the practical application of Saint-Venant's principle to sandwich structures.

The papers (vi, vii) are concerned with end effects in *functionally graded materials* (FGM's). Such applications arise in bonding ceramics to metal armor-plates, for example. The effects of such inhomogeneities on stress decay are analyzed in (vi) and (vii).

It has been known for some time that certain radial anisotropies in some linear elasticity problems can give rise to stress singularities which are absent in the corresponding isotropic problems. Recently related issues were examined by other authors in the context of plane strain axisymmetric deformations of a hollow circular cylindrically anisotropic linearly elastic cylinder under uniform external pressure, an anisotropic analog of the classic isotropic Lame problem. In the isotropic case, as the external radius increases, the stresses rapidly approach those for a traction-free cavity in an infinite medium under remotely applied uniform compression. However, it has been shown that this does *not* occur when the cylinder is even slightly anisotropic. In (iii), we provide further elaboration on these issues. For the externally pressurized hollow cylinder (or disk), it is shown that for radially orthotropic materials, the maximum hoop stress occurs always on the inner boundary (as in the isotropic case) but that the stress concentration

factor is infinite. For circumferentially orthotropic materials, if the tube is sufficiently thin, the maximum hoop stress always occurs on the inner boundary whereas for sufficiently thick tubes, the maximum hoop stress occurs at the outer boundary. For the case of an *internally* pressurized tube, the anisotropic problem does *not* give rise to such radical differences in stress behavior from the isotropic problem. Such differences do, however, arise in the problem of an anisotropic disk, in plane stress, rotating at a constant angular velocity about its center, as well as in the three-dimensional problem governing radially symmetric deformations of anisotropic externally pressurized hollow spheres. The anisotropies of concern here do arise in technological applications such as the processing of fiber composites as well as the casting of metals.

5. TECHNOLOGY TRANSFER

The results of this research are being widely utilized in the technical literature on composite materials, with technology transfer to such areas as composite design and materials testing. The attached summary of technology transfer to the Boeing Commercial Airplane Group attests to the applicability of the results. Dr. Horgan visited the Boeing Company in Seattle on May 25, 1995 to further this important area of technology transfer. He presented a 1-hour lecture entitled *End Effects in Composite Structures* to the Boeing group. This interaction continues (see letter attached, July, 1996). We have also interacted extensively with Dr. M. P. Nemeth, NASA Langley, Structural Mechanics Division. Dr. Nemeth served as a committee member for the Ph.D. dissertation defense of Ms. Sarah C. Baxter on April 1995. Her dissertation was entitled "Saint-Venant end effects for anti-plane shear deformations of sandwich structures." Dr. Baxter is currently an assistant Professor in the Department of Mechanical Engineering, University of South Carolina, Columbia, S.C.

Professor Horgan has presented numerous invited colloquium lectures on his AFOSR supported work at DoD laboratories and several universities in the U.S. and Europe (Italy, France) and at National and International Meetings. He serves on the editorial boards of five major journals, namely, *Applied Mechanics Reviews, International J. of Nonlinear Mechanics, J. of Elasticity, SIAM J. on Applied Mathematics, Mathematics and Mechanics of Solids* , the last of which he is a founding editor. He is a fellow of ASME and the American Academy of Mechanics. Professor Horgan was elected to the Board of Directors, Society of Engineering Science, in 1993, and he was re-elected in 1996.

6. STUDENTS SUPPORTED

1. Sarah C. Baxter (Ph.D. Degree, 1995)

A technical description of Dr. Baxter's work is given in the Scientific Progress and Accomplishments section. Her work under the support of the AASERT Program resulted in three refereed journal publications and two refereed proceedings articles. Dr. Baxter spent the period 1995-1997 as a Post-Doctoral Fellow, Dept. of Civil Engineering and Applied Mechanics, University of Virginia. She has presented several lectures on her work at conferences and university colloquia. She is currently an Assistant Professor in the Department of Mechanical Engineering, University of South Carolina, Columbia.

2. Monica R. Scalpato (M.S. Degree, 1997)

Ms. Scalpato's Masters Dissertation is concerned with edge effects in functionally graded materials (FGM's). The work has been published (ref (vi) above) in the *Journal of Elasticity.* A technical description of the work is given in the Scientific Progress and Accomplishments section. Ms. Scalpato is currently employed as Senior Information Technology Consultant, Ernst and Young LLP, Boston, Mass.

3. Alice M. Chan (M. S. Degree, 1999)

Ms. Chan has completed most of the course requirements for the M. S. Degree. She is preparing a Master's Dissertation entitled " Boundary-Value Problems for Functionally-Graded Materials " . One technical paper (ref (vii) above) has been submitted for publication, with others to follow.

4. Adam J. Lewandowski (B. S. Degree, 1998)

Mr. Lewandowski was supported on an hourly salary basis during his senior undergraduate year 1997/98 to assist in computing tasks associated with the research. This research experience proved very beneficial to him as he decided to continue with graduate studies at Johns Hopkins University where he was awarded an Owen Fellowship in Physics starting in September 1998.

A Technology Transfer Example

A Boeing/NASA Advanced Technology Composite Aircraft Structures (ATCAS) Program has been active since 1989. The primary objective of this program is to:

> "Develop an integrated technology (manufacturing & structures) and demonstrate a confidence level that permits cost-and weight-effective use of advanced composite materials in primary structures of aircraft with the emphasis on pressurized fuselages."

In this program, a section of a widebody aircraft (244" dia) just aft of the wing/body intersection is being analyzed by the Boeing Commercial Airplane Group in Seattle, Washington. Sandwich structures are being used for the side and keel of this section. The particular structures consist of Hercules' AS4/8552 for the skin and Hexcel's HRP honeycomb core (see next page for details on layup etc.). Compression testing of laminate coupons indicate the need to incorporate Saint-Venant end effects in interpretation of the test data. The work of the PI's is being utilized in this effort. One of the P.I's (C. O. H.) visited the Boeing Group in Seattle on July 1, 1994 and on June 1/2, 1995 to consolidate this interaction. Collaborative research with the Boeing scientists (Dr. W. A. Avery, coordinator) is being initiated.. One objective is to develop a systematic testing program to be carried out by Integrated Technologies, Inc. (Intec), Bothell, WA, under subcontract to Boeing. Preliminary tests by Intec have indicated problems due to end effects in the sandwich panels under investigation. It is anticipated that the results obtained in our research program will have direct application to these problems. In fact, the interaction with the Boeing/Intec mechanics and materials group is providing additional motivation and stimulus to our efforts in understanding the extent of Saint-Venant end effects in advanced composite materials and structures.

Table 1 Material Types

Panel ID	Skin Material	Form	No of Plies	Nominal Ply Thickness *(in)*	Layup ID	Core Type	Core Thick-ness *(in)*
AK7	8-256	Tow	12	0.0080	Keel 1	HRP-3/16-8.0	0.75
AK8	8-256	Tow	12	0.0080	Keel 1	HRP-3/16-8.0 & TPC-3/16/5.5	0.75
AK10a	AS4/8552	Tape	12	0.0073	Keel 1	HRP-3/16-8.0	0.75
AK10b	AS4/8552	Tape	12	0.0073	Keel 3	HRP-3/16-8.0	0.75
AK10c	AS4/8552	Tow	12	0.0073	Keel 1	HRP-3/16-8.0	0.75
AK10d	AS4/8552	Tow	12	0.0073	Keel 1/ Keel 3	HRP-3/16-8.0	0.75

Table 2 Layups

Layup ID	Number of Plies	Ply Orientation
Keel 1	12	[45/0/-45/90/0/-45/45/0/90/-45/0/45]
Keel 3	12	[30/-30/0/90/0/-45/45/0/90/0/-30/30]

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July 5, 1996
BYH20-BFB-L96-043

Dr. Arje Nachman
Program Director, Applied Analysis
AFOSR, 110 Duncan Ave., Suite B115
Bolling AFB
Washington, DC 20332-0001

BOEING

Dear Dr. Nachman,

I few days ago Cornelius Horgan from the University of Virginia asked me to write you and explain the relevance of his work to Boeing. I am happy to do so. This letter is the response to that request.

In 1989 Boeing started work on the Advanced Composite Technology Aircraft Structure (ATCAS) program, which is funded by NASA's Advanced Composite Technology (ACT) initiative. In this program Boeing has been developing the materials, structures, and manufacturing technology for a composite fuselage for a widebody aircraft. The goal is to design the structure such that there is significant savings in both cost and weight.

Early in the program we identified sandwich structure as having a high potential to save cost because the tooling and manufacturing processes for skin-stringer structures are expensive. Consequently we baselined sandwich construction for the fuselage keel and side panels.

When we started collecting our structural database we compression tested several solid laminate and sandwich coupons, many of them with holes. The failure loads were higher than expected, and we realized that we may have not been getting the full stress concentrations at the edges of the holes. This was confirmed through some photoelastic analyses of additional coupons. Part of the problem was test method related. But a review of some of Dr. Horgan's work helped us understand how St. Venant's effects are different for anisotropic and sandwich structures. His work became useful in guiding us in sizing test coupons based on degree of anisotropy and the particular configuration of the sandwich structure. Essentially, we found that we needed a longer specimen in order to get uniform load into the test coupon. Dr. Horgan's analyses helped us quantify that increase in length.

I appreciate Dr. Horgan's effort to keep in touch with Boeing and solicit ideas for research. I find him very "customer oriented". His work has been helpful to us. If you have any questions please feel free to contact me.

Regards,

William Avery, Ph.D.
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Edge Effects in Composite Structures

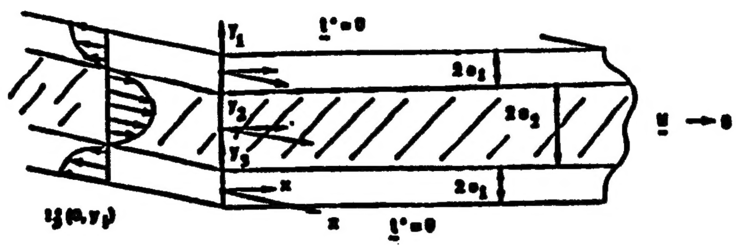
C. O. Horgan and J. G. Simmonds
School of Engineering & Applied Science
University of Virginia

Assumptions

I. Sandwich Structures

- Anti-plane shear
- Linear elasticity

$\delta = \frac{\mu_1}{\mu_2}$, f = volume fraction, $\bar{d}$= decay length



Results

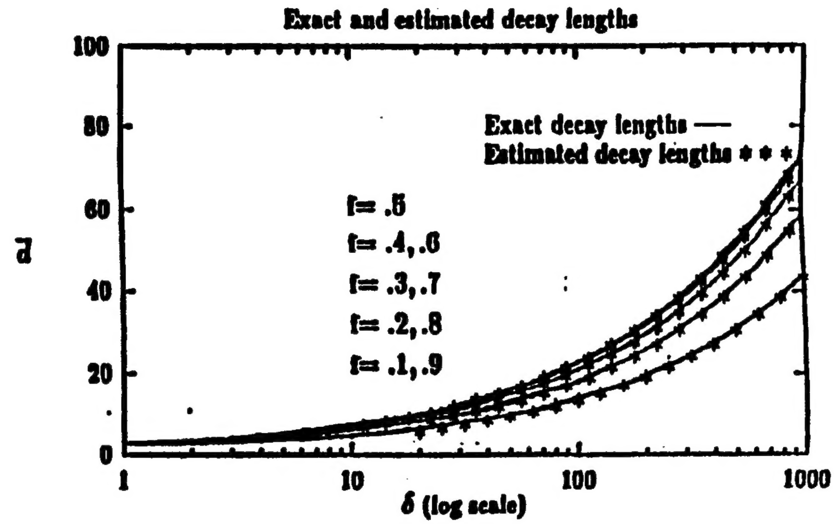


Fig. 2. Scaled decay length vs. δ, for various f

Decay length for semi-infinite sandwich strips subjected to self-equilibrated end loads.

The characteristic decay length (i.e., the distance over which end effects decay to 1% of their end values) versus a nondimensional material parameter (the ratio of face to core shear modulus) is plotted in Fig. 2. In Fig. 3, the plot is for varying volume fraction and fixed δ. The decay length is seen to be *smallest* for a homogeneous isotropic material, $\delta = 1$, as shown in Fig. 2. This decay length is approximately equal to the width of the strip. From Fig. 3, the decay length for fixed δ, is seen to be *largest* at a volume fraction f=.5. These figures (and the associated asymptotic formulas) can be used directly in the design process for sandwich structures.

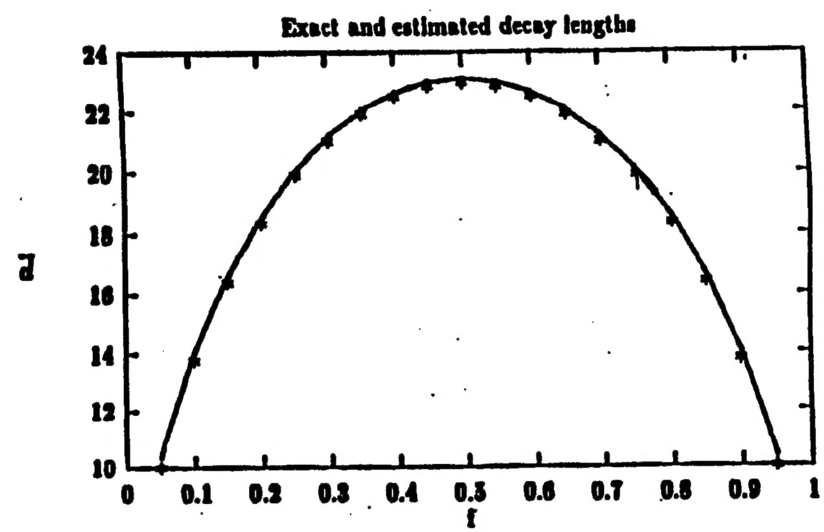


Fig. 3. Scaled decay length vs. volume fraction $(\delta = 100)$

Saint-Venant End Effects for Anti-Plane Shear Deformations of Sandwich Structures

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A Dissertation

Presented to

the Faculty of the School of Engineering and Applied Science

University of Virginia

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy (Applied Mathematics)

by

Sarah Collins Baxter

Abstract

The purpose of this research is to further investigate the effects of material inhomogeneity and the combined effects of material inhomogeneity and anisotropy on the decay of Saint-Venant end effects. Saint-Venant decay rates for self-equilibrated edge loads in symmetric sandwich structures are examined in the context of anti-plane shear for linear anisotropic elasticity. The problem is governed by a second-order, linear, elliptic partial differential equation with discontinuous coefficients. The most general anisotropy consistent with a state of anti-plane shear is considered, as well as a variety of boundary conditions. Anti-plane or longitudinal shear deformations are one of the simplest classes of deformations in solid mechanics. The resulting deformations are completely characterized by a single out-of-plane displacement which depends only on the in-plane coordinates. They can be thought of as complementary deformations to those of plane elasticity. While these deformations have received little attention compared with the plane problems of linear elasticity, they have recently been investigated for *anisotropic* and *inhomogeneous* linear elasticity. In the context of linear elasticity, Saint-Venant's principle is used to show that self-equilibrated loads generate local stress effects which quickly decay away from the loaded end of a structure. For homogeneous *isotropic* linear elastic materials this is well-documented. Self-equilibrated loads are a class of load distributions that are statically equivalent to zero, i.e., have zero resultant force and moment. When Saint-Venant's principle is valid, pointwise boundary conditions can be replaced by more tractable resultant conditions. It is shown in the present study that material inhomogeneity significantly affects the practical application of Saint-Venant's principle to sandwich structures.

SAINT - VENANT DECAY RATES FOR AN ISOTROPIC INHOMOGENEOUS LINEARLY ELASTIC SOLID IN ANTI - PLANE SHEAR

A THESIS

Presented to

the faculty of the School of Engineering and Applied Science

The University of Virginia

In partial fulfillment

of the requirements for the degree of

MASTER OF SCIENCE

Institute of Applied Mathematics and Mechanics

by

Monica R. Scalpato

May 1997

ABSTRACT

The purpose of this research is to investigate the effects of material inhomogeneity on the decay of Saint - Venant end effects in linear isotropic elasticity. This question is addressed within the context of anti - plane shear deformations of an inhomogeneous isotropic elastic solid. The mathematical issues involve the effects of spatial inhomogeneity on the decay rates of solutions to Dirichlet or Neumann boundary - value problems for a second - order linear elliptic partial differential equation with variable coefficients on a semi - infinite strip. The elastic coefficients are assumed to be smooth functions of the transverse coordinate. The *estimated rate* of exponential decay with distance from the loaded end (a lower bound for the exact rate of decay) is characterized in terms of the smallest positive eigenvalue of a Sturm - Liouville problem with variable coefficients. Analytic lower bounds for this eigenvalue are used to obtain the desired estimated decay rates. Numerical techniques are also employed to assess the accuracy of the analytical results.

DISTRIBUTION LIST

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